Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Thursday 24 May 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40685A

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5,$$

giving each term in its simplest form.

(4)

---

2. Find the values of x such that

$$2 \log_3 x - \log_3(x - 2) = 2$$

(5)

3.

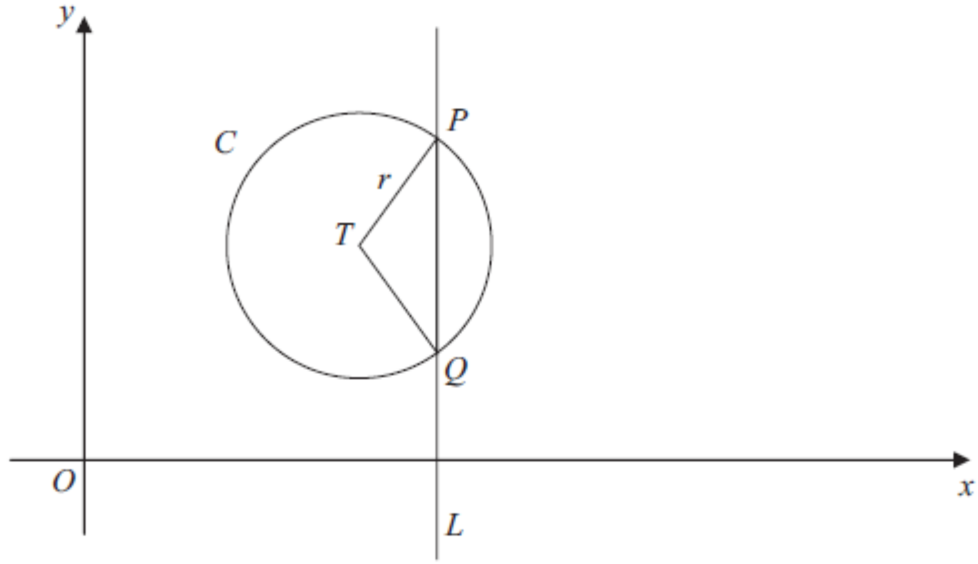

Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0.$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

(d) find the perimeter of the sector PTQ.

(3)

---

4. $$f(x) = 2x^3 - 7x^2 - 10x + 24.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

5.

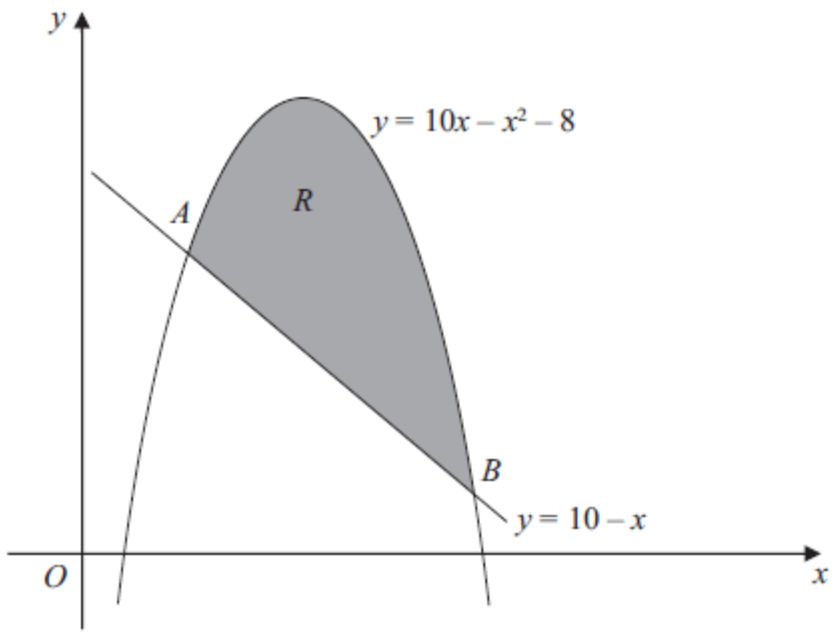

Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$.

The line and the curve intersect at the points A and B, and O is the origin.

(*a*) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(*b*) Calculate the exact area of R.

(7)

---

6. (*a*) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x \leq 180°$,

$$\tan 2x = 5 \sin 2x,$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

---

7. $$y = \sqrt{(3^x + x)}$$

(*a*) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(*b*) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x + x)}\ \mathrm{d}x .$$

You must show clearly how you obtained your answer.

(4)

8.

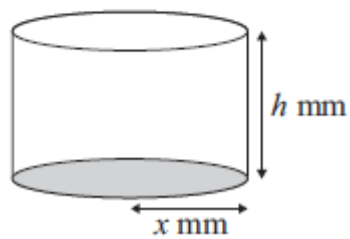

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(*a*) express h in terms of x,

(1)

(*b*) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$.

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(*c*) Use calculus to find the value of x for which A is a minimum.

(5)

(*d*) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(*e*) Show that this value of A is a minimum.

(2)

---

9. A geometric series is $a + ar + ar^2 + ...$

(*a*) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(*b*) the common ratio,

(2)

(*c*) the first term,

(2)

(*d*) the sum to infinity.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Summer 2012
6664 Core Mathematics 2
Mark Scheme

Question number	Scheme	Marks
1	$\left[(2-3x)^5\right] = \ \ldots \quad +\binom{5}{1}2^4(-3x)+\binom{5}{2}2^3(-3x)^2+..,\ \ \ldots\ldots$	M1
	$= 32,\ -240x,\ +720x^2$	B1, A1 , A1
		Total 4
Notes	**M1**: The **method** mark is awarded for an attempt at Binomial to get the second and/or third term – need **correct** binomial coefficient combined **with correct power of *x*.** Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for $^5C_1$ and $^5C_2$, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including x is correct. **B1:** must be simplified to 32 (writing just 2^5 is **B0). 32** must be the only constant term in the final answer- so 32 +80 - 3*x*+80 +9 x^2 is B0 but may be eligible for M1A0A0 . **A1: is cao and is for –**240 x. (not +-240x) The x is required for this mark **A1:** is c.a.o and is for $720x^2$ (can follow omission of negative sign in working) A list of correct terms may be given credit i.e. series appearing on different lines Ignore extra terms in x^3 and/or x^4 (isw)	
Special Case	Special Case*: Descending powers* of x would be $(-3x)^5+2\times5\times(-3x)^4+2^2\times\binom{5}{3}\times(-3x)^3+..$ i.e. $-243x^5+810x^4-1080x^3+..$ This is a misread but award as s.c. M1B1A0A0 if completely "correct" or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x	
Alternative Method	**Method 1:** $\left[(2-3x)^5\right]=2^5(1+\binom{5}{1}(-\frac{3x}{2})+\binom{5}{2}(\frac{-3x}{2})^2+..\ )$ is **M1B0A0A0 {** The M1 is for the expression **in the bracket** and as in first method– need **correct** binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors} – answers must be simplified to $=32,-240x,+720x^2$ for full marks (awarded as before) $\left[(2-3x)^5\right]=2(1+\binom{5}{1}(-\frac{3x}{2})+\binom{5}{2}(\frac{-3x}{2})^2+..\ )$ would also be awarded **M1B0A0A0** **Method 2: Multiplying out** : B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4	

Question number	Scheme	Marks
2	$2\log x = \log x^2$	B1
	$\log_3 x^2 - \log_3(x-2) = \log_3 \dfrac{x^2}{x-2}$	M1
	$\dfrac{x^2}{x-2} = 9$	A1 o.e.
	Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
	$x = 3$, $x = 6$	A1
		Total 5
Notes	**B1** for **this** correct use of power rule (may be implied) **M1:** for **correct** use of subtraction rule (or addition rule) for logs N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \dfrac{x}{x-2}$ is **M0** **A1**. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.) **M1** for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics) **A1** for these t**wo** correct answers	
Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1, *so* $x^2 = 3^{2+\log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1 Here M1 is for **complete** method i.e.correct use of powers after logs are used correctly	
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but **the statement is M0** and so leads to no further marks $2\log_3 x - \log_3(x-2) = 2$ *so* $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \dfrac{x}{x-2} = 1$ can earn M1 for *correct* subtraction rule following error, but no other marks	
Special Case	$\dfrac{\log x^2}{\log(x-2)} = 2$ leading to $\dfrac{x^2}{x-2} = 9$ and then to x =3, x =6, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This *recovery* after uncorrected error is very common] Trial and error, Use of a table or just stating answer **with both** x=3 and x=6 should be awarded B0M0A0 then final M1A1 i.e. 2/5	

Question number	Scheme	Marks
3 (a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$ Obtain $(x - 10)^2$ **and** $(y - 8)^2$ Centre is $(10, 8)$. N.B. This may be indicated on **diagram only** as (10, 8)	M1 A1 A1 (3)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 =)"100"+"64"-139$ $r = 5$ * (this is a printed answer so need one of the above two reasons)	M1 **A1** **(2)**
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$ e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y=$ *or* $13^2 + y^2 - 20\times13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$ $y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**M1** **A1, A1** (3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275) Perimeter $PTQ = 2r$ + their **arc** PQ (Finding perimeter of triangle is M0 here) = 19.275 or 19.28 or 19.3	**M1** **M1** **A1** (3)
		11 marks
<u>Alternatives</u> (a) OR	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$ Centre is $(-g, -f)$, **and so centre is** $(10, 8)$. *Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1, A1 M1 A1 A1 (3)
(b) OR	*Method 2*: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)"100"+"64"-139$ $r = 5$ * *Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$ $r = 5$ *	M1 A1 **M1** **A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate $"8 \pm h"$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**
Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be <u>implied</u> by $(\pm10, \pm8)$ **. Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 = "100"+"64"-139$ (not 139 – "100" –" 64") or for using equation of circle in $(x \pm 10)^2 + (y \pm 8)^2 = k^2$ form to identify $r=$ **3rd A1 $r = 5$ (NB This is a given answer so should follow $k^2 = 25$ or $r^2 = 100 + 64 - 139$)** **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

Question number	Scheme	Marks
4 (a)	$f(-2) = 2.(-2)^3 - 7.(-2)^2 - 10.(-2) + 24$ $= 0$ so $(x+2)$ is a factor	M1 A1 (2)
(b)	$f(x) = (x+2)(2x^2 - 11x + 12)$ $f(x) = (x+2)(2x-3)(x-4)$	M1 A1 dM1 A1 (4)
		6 marks
Notes (a) (b)	**M1** : Attempts f(± 2) (Long division is M0) **A1** : is for =0 **and conclusion** **Note**: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a preamble, eg: "If $f(-2) = 0$, $(x + 2)$ is a factor…." (Not just f(-2)=0) **1st M1**: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or *Alternative Method* : **1st M1**: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c **1st A1**: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] **2nd M1**: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors** **2nd A1**: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}, x = \frac{3}{2}, 4$, and not list all three factors. Award these responses M1A1M0A0. Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1 $f(x) = (x+2)(2x-3)(x-4)$ **or** $f(x) = 2(x+2)(x-1.5)(x-4)$ **o.e. is full marks** $f(x) = (x+2)(x-1.5)(x-4)$ **loses last A1**	

Question number	Scheme		Marks
Method 1 5 (a)	Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic	Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$	Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$	M1
	Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)	Obtains $y = 8, y = 1$ (may be on diagram)	A1
	Substitutes their x into a given equation to give $y =$ (may be on diagram)	Substitutes their y into a given equation to give $x=$ (may be on diagram or in part (b))	M1
	$y = 8, y = 1$	$x = 2, x = 9$	A1 (5)
(b)	$\int (10x - x^2 - 8)\,dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{+ c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]_2^9 = (......) - (......)$		dM1
	$= 90 - \frac{4}{3} = 88\frac{2}{3}$ or $\frac{266}{3}$		
	Area of trapezium $= \frac{1}{2}(8 + 1)(9 - 2) = 31.5$		B1
	So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks

Notes (a)

First **M1:** See scheme Second **M1**: See notes relating to solving quadratics
Third **M1**: This may be awarded if one substitution is made
Two correct Answers following tables of values, or from Graphical calculator are 5/5
Just one pair of correct coordinates – no working or from table is M0M0A0M1A0

(b)

M1: $x^n \rightarrow x^{n+1}$ for any one term.
1st A1: at least two out of three terms correct **2nd A1**: All three correct
dM1: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round
(NB: If candidate **changes all signs** to get $\int (-10x + x^2 + 8)\,dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{+ c\}$ This is M1 A1 A1
Then uses limits dM1 and trapezium is B1
Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0)
B1: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2} \times 8 \times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6]
M1: Their Area under curve – Their Area under line (if integrate both need same limits)
A1: Accept 57.16recurring but not 57.16

PTO for Alternative method

Method 2 for (b)	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ $\int_2^9 -x^2 + 11x - 18\,dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (......) - (......)$	3rd M1 (in (b)): Uses difference between two functions in integral. M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of these three simplified terms Correct integration. (Ignore + c). Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	M1 A1 A1 dM1
	This mark is implied by final answer which rounds to 57.2 ***See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:*** $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao		B1 M1 A1 (7)
Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao		M1A1A1 DM1 B1 M1 A1 (7)
Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ only earns final M mark		

Question number	Scheme		Marks
6(a)	States or uses $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$		M1
	$\dfrac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1 - 5\cos 2x) = 0$ *		A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) **or** $2x = 281.54$ (or 281.6)		M1
	$x = 39.2$ (or 39.3), 140.8 (or 141)		A1, A1 (5)
			7 marks
Notes	(a) **M1**: Statement that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$. **A1**: the answer is given so all steps should be given. N.B. $\sin 2x - 5\sin 2x\cos 2x = 0$ or $-5\sin 2x\cos 2x + \sin 2x = 0$ or $\sin 2x(\dfrac{1}{\cos 2x} - 5) = 0$ o.e. **must be seen** and be followed by printed answer for A1 mark $\sin 2x = 5\sin 2x\cos 2x$ is not sufficient. (b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions **M1**: This mark for one of the two statements given (must relate to $2x$ not just to x) **A1, A1**: first A1 for 39.2, second for 140.8 *Special case* solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141 Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14) Excess answers **in range** lose last A1 Ignore excess answers outside range. All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here		

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7 (a)

(b)</td><td>

x	0	0.25	0.5	0.75	1
y	1	1.251	**1.494**	**1.741**	2

$\frac{1}{2}\times 0.25,\ \left\{(1+2)+2\left(1.251+1.494+1.741\right)\right\}$ o.e.
$=1.4965$</td><td>B1, B1
(2)
B1, M1,A1 ft

A1
(4)
6 marks</td></tr>
<tr><td>Notes</td><td colspan="2">(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B0)
Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b) B1: Need ½ of 0.25 or 0.125 o.e.
M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however)
x values: M0 if values used in brackets are x values instead of y values

A1ft follows their answers to part (a) and is for {correct expression}
Final A1: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table)
Separate trapezia may be used : B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1ft if it is all correct)
e.g.. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is M1 A0
equivalent to missing one term in { } in main scheme

Special Case: Bracketing mistake: i.e. 0.125(1+2) +2(1.251+1.494+1.741)
scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given).
Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review

Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0 B1M1A1ft then A1 (lose 1 mark)

NB Bracket is 11.972</td></tr>
</table>

Question number	Scheme		Marks
8 (a)	$(h=)\frac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60 \div \pi x^2$		B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines		B1
	Either $(A)=2\pi x^2+2\pi x\left(\frac{60}{\pi x^2}\right)$ or As $\pi xh=\frac{60}{x}$ then $(A=)2\pi x^2+2\left(\frac{60}{x}\right)$		M1
	$A=2\pi x^2+\left(\frac{120}{x}\right)$ *		A1 cso (3)
(c)	$(\frac{dA}{dx})=4\pi x-\frac{120}{x^2}$ or $=4\pi x-120x^{-2}$		M1 A1
	$4\pi x-\frac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)		M1
	$x=\sqrt[3]{\frac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)		dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\frac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)		M1, A1 (2)
(e)	**Either** $\frac{d^2A}{dx^2}=4\pi+\frac{240}{x^3}$ and sign considered (May appear in (c))	**Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1
	which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c))	Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	A1 (2)
			13 marks

Notes

(a) **B1**: This expression must be correct and in part (a) $\frac{60}{\pi r^2}$ is B0

(b) **B1:** Accept any equivalent correct form – may be on two or more lines.

M1 : substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh

A1: There should have been no errors in part (b) in obtaining this printed answer

(c) **M1:** At least one power of x decreased by 1 **A1** accept any equivalent correct answer

M1: Setting $\frac{dA}{dx}$=0 and finding a value for x^3 (x^3 = may be implied by answer). Allow $\frac{dy}{dx}=0$

dM1: Using **cube root** to find x

A1 : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark

(d) **M1 :** Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only**

(e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered

A1: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)

Question	Scheme	Marks
9 (a)	$(S_n =)\ a + ar + (ar^2) + \ldots \quad + ar^{n-1}$ **and** $rS_n = ar + ar^2 + (ar^3)\ldots \quad + ar^n$	M1
	$S_n - rS_n = a - ar^n$	M1
	$S_n(1-r) = a(1-r^n)$	dM1
	And so result $S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1 (4)
(b)	Divides one term by other (either way) to give $r^2 = \ldots$ then square roots to give $r =$ **Or:** (*Method 2)* Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term	M1
	$r^2 = \dfrac{1.944}{5.4}$, $r = 0.6$ (ignore – 0.6) $r = 0.6$ (ignore – 0.6)	A1 (2)
(c)	Uses $5.4 \div r^2$ or $1.944 \div r^4$, to give $a =$ $a = 15$	M1, A1ft (2)
(d)	Uses $S = \dfrac{15}{1-0.6}$, to obtain 37.5	M1A1 ,A1 (3)
		11 marks
Notes Special Case	**(a) M1:** Lists **both** of these sums (S_n =) may be omitted, $r\,S_n$ (or rS) must be stated 1st two terms must be correct in each series. Last term must be ar^{n-1} or ar^n in first series and the corresponding ar^n or ar^{n+1} in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms **M1:** Subtracts series for rS from series for S (or other way round) to give RHS $= \pm(a - ar^n)$. This may have been obtained by following a pattern**. If wrong power stated on line 1 M0 here**. (Ignore LHS)M0M0M0A0 d**M1**: Factorises both sides correctly– must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) **A1**: completes the proof with **no errors** seen No errors seen: **First line absolutely correct**, omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving **sigma notation**, or any **proofs by induction** to team leader. Also attempts which **begin with the answer and work backwards**. **(b) M1:** Deduces r^2 by dividing either term by other and attempts square root **A1:** any correct equivalent for r e.g. 3/5 Answer only is 2/2 (*Method 2*) Those who find fourth term must use $\sqrt{ab}$ and **not** $\frac{1}{2}(a+b)$ then must use it in a division with given term to obtain $r =$ **(c) M1:** May be done in two steps or more e.g. $5.4 \div r$ then divided by r again **A1ft:** follow through their value of r . Just $a = 15$ **with no wrong working** implies M1A1 (d) **M1**: States sum to infinity formula with values of a and r found earlier, provided $\lvert r\rvert < 1$ **A1** : uses 15 and 0.6 (or 3/5) (This is **not** a ft mark) **A1**: 37.5 or exact equivalent	
Common errors	(i) Fraction inverted in (b) $r^2 = \dfrac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. 3/7 (ii) Uses $r = 0.36$: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. 3/7 (iii) Uses $ar^3 = 5.4$, $ar^5 = 1.944$ Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e.3/7	